CALCULATION OF CONTACT TEMPERATURES GENERATED BY THE ROTATION OF A SHAFT IN A BEARING

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 2, pp. 135–137, 1968

The problem of the distribution of contact stresses resulting from the interaction between a journal and its bearing was considered in [1]. This paper deals with the problem of temperature distribution in the area of contact of a rotating cylindrical shaft and a bearing. The process is assumed to be stabilized.

The problem reduces to an integral equation with respect to the contact temperature at the shaft surface.

An approximate method is proposed for solving the integral equation which had permitted the derivation of a simple approximate formula for the contact temperature within any range of variation of the parameters of this problem.

1. An infinitely long shaft rotates in its bearing [1] at a uniform angular velocity ω. The area of contact between the shaft and the bearing is assumed to be a cylindrical rectangle with $|z| \le L/2$ and $|\theta| \le \theta_0$.

If $q(\theta, z)$ are the normal contact stresses, f is the coefficient of friction, R is the shaft radius, and c is the heat equivalent of mechanical work, the quantity of heat absorbed by the shaft is defined [2] by the expression

$$Q = c\alpha\omega fRq(\theta, z).$$

Here α is the coefficient of heat distribution between the shaft and the bearing. This coefficient is, strictly speaking, a function of the contact point (z, θ). We consider it as constant.

2. The boundary condition corresponding to the physical picture (a given heat flux through the contact surface) in the contact region is of the form

$$\frac{\partial T}{\partial r} = \frac{c\alpha\omega}{K} fRq(\theta, z), \quad r = R, \quad |\theta| \leqslant \theta_0, \quad |z| \leqslant L/2. \tag{1}$$

Here, T is the shaft temperature and K is the coefficient of the shaft heat conductivity.

We assume that, in the region $|\theta| \ge \theta_0$, $|z| \le L/2$, the heat flux is absent, i.e.,

$$\partial T / \partial r = 0, \qquad r = R. \tag{2}$$

Heat exchange with the surrounding medium is assumed along the part of the boundary where $|z| > L/2$:

$$\gamma \partial T / \partial r + T = 0, \qquad \gamma = K / H. \tag{3}$$

Here, H is the heat transfer coefficient of the shaft.

Temperature T is defined by the heat-transfer equation

$$\frac{1}{\varkappa}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}, \tag{4}$$

where $\varkappa$ is the coefficient of thermal diffusivity.

3. The thermal process is assumed to be stable. We introduce the dimensionless parameters ρ, l, ξ, and the averaged temperature

$$u = \frac{1}{2\pi}\int_0^{2\pi} T d\theta, \quad \rho = \frac{r}{R}, \quad l = \frac{L}{R}, \quad \zeta = \frac{z}{R}. \tag{5}$$

As the result, we obtain the following mixed boundary-value problem with respect to the function $u = u(\xi)$

$$\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + \frac{\partial^2 u}{\partial \zeta^2} = 0; \tag{6}$$

$$\frac{\partial u}{\partial \rho} + \lambda u = 0, \quad |\zeta| > \frac{l}{2}, \quad \rho = 1;$$

$$\frac{\partial u}{\partial \rho} = \alpha M(\zeta), \quad |\zeta| \leqslant \frac{L}{2}, \quad \rho = 1; \tag{7}$$

$$M(\zeta) = \frac{c\omega f R^2}{K 2\pi}\int_{-\theta_0}^{\theta_0} q(\theta, \zeta)\, d\theta. \tag{8}$$

Using the Fourier transformation, we reduce the boundary-value problem (6), (7) to the equation

$$\int_{-a}^{a} k(x - \xi)\, g(\xi)\, d\xi = 2\pi\alpha M(x) \quad |x| \leqslant a, \quad a = l/2; \tag{9}$$

$$k(t) = 2\int_0^\infty \frac{\eta I_1(\eta)\cos\eta t\, d\eta}{\eta I_1(\eta) + \lambda I_0(\eta)}, \quad u(x)|_{\zeta=1} = \lambda^{-1}[g(x) - \alpha M],$$

$$\lambda = \frac{RH}{K}. \tag{10}$$

An exact solution of Eq. (9) is not possible. The properties of its solution, have, however, been the subject of detailed analysis [3]. It has been, in particular, established that, when the right-hand side of Eq. (9) is a bounded function, its solution is also bounded.

4. We shall solve, instead of Eq. (9), a certain approximate equation derived by the substitution of an approximation for the exact kernel. We shall use the following approximate formula:

$$\frac{\eta I_1(\eta)}{\eta I_1(\eta) + \lambda I_0(\eta)} \approx \frac{\eta^2(\eta^2 + c_2)}{\eta^4 + (c_1 + \lambda c_3)\eta^2 + \lambda c_2} = N(\eta), \quad \eta \geqslant 0;$$

$$c_1 = 38.37, \quad c_2 = 76.74, \quad c_3 = 11.45. \tag{11}$$

The relative error of this approximation does not exceed 3% for all values of the parameter $\eta > 0$, and for $0 < \lambda \le 0.3$. (If one takes into account that in the case of metallic shafts $H/K \sim 0.1$–0.4 (in m^{-1}), it becomes clear that this range of λ-variation comprises the most significant range of shaft radii.)

Thus, the problem reduces to the solution of the integral equation

$$\int_{-a}^{a} k^*(x - \xi)\, g^*(\xi)\, d\xi = 2\pi\alpha M(x), \qquad |x| \leqslant a,$$

$$k^*(t) = 2\int_0^\infty N(\eta)\cos\eta t\, d\eta. \tag{12}$$

It can be shown that, in the case considered here, the relative error of solutions for Eqs. (9) and (12) is on the order of the error of approximation (11).

Equation (12) was analyzed by Simonenko in [4]. Here its solution is derived in a simpler manner based on the method presented in [3].

Using the results of [1], we assume the right-hand side of Eq. (12) to be independent of x. Its unique solution may then be presented in the form

$$g^*(x) = M\alpha\,[A_1 + 2A_2 e^{-a\sqrt{c_1}}\,\mathrm{ch}\, a\sqrt{c_1}\, x + A_3(a^2 + x^2)]. \tag{13}$$

Here, A_k are unknown coefficients independent of x and are determined from the system of algebraic equations

$$-\frac{2\pi c_1}{\lambda c_2}A_3 = 2\pi, \qquad \sigma_k A_1 + \tau_k A_2 = \delta_k,$$

$$\delta_k = -\frac{\lambda c_2 M}{c_1}\left(\frac{5}{\zeta_k^3} + \frac{4a}{\zeta_k^2} + \frac{4a^2}{\zeta_k}\right) \quad k = 1, 2;$$

$$\sigma_k = -\frac{1}{\zeta_k}, \qquad \tau_k = \frac{1}{\sqrt{c_1} - \zeta_k} - \frac{e^{-2a\sqrt{c_1}}}{\sqrt{c_1} + \zeta_k}, \tag{14}$$

where $i\zeta_1$ and $i\zeta_2$ are zeros of polynomial $t^4 + (c_1 + \lambda c_3)t^2 + \lambda c_2$ lying in the upper half-plane

$$\zeta_{1,2} = \left[\frac{c_1 + \lambda c_3}{2} \pm \left(\frac{(c_1 + \lambda c_3)^2}{4} - \lambda c_3\right)^{1/2}\right]^{1/2}, \tag{15}$$

From (14), we find that

$$A_1 = \frac{\delta_1\tau_2 - \delta_2\tau_1}{\sigma_1\tau_2 - \sigma_2\tau_1}, \quad A_2 = \frac{\sigma_1\delta_2 - \sigma_2\delta_1}{\sigma_1\tau_2 - \sigma_2\tau_1}, \quad A_3 = -\frac{\lambda c_2}{c_1}. \qquad (16)$$

The final expression for the averaged temperature in the contact region is

$$u(\zeta) = \frac{M\alpha}{\lambda}\,[g^*(\zeta) - 1]. \qquad (17)$$

5. Let us consider the problem of contact temperature generated by the rotation of a shaft in a polymer bearing [1]. In this problem $\lambda \sim \sim 0.03$, hence, formula (17) is substantially simplified. Expanding coefficients A_1, A_2, and A_3 in a series in λ, and limiting these to their first terms, we obtain

$$u(z) = \frac{2\alpha\theta_0 f\omega c R G\Delta(1-\nu)}{\pi\lambda K\varepsilon(1-2\nu)}\left(\frac{\operatorname{tg}\theta_0}{\theta_0} - 1\right)\times$$

$$\times\left[4 + \frac{21.9}{\sqrt{\lambda}}\,e^{-3.1L/R}\operatorname{ch} 3.1z/R - 0.5\lambda\,\frac{L^2 + 4z^2}{R^2}\right]. \qquad (18)$$

Parameters G, Δ, ν, and ε appearing in formula (18) have been borrowed from [1] and denote, respectively, the following: the shear modulus of the bearing material, the bearing eccentricity, the Poisson coefficient of the bearing material, and the relative thickness of the polymer bearing.

Derived formula (18) is convenient for engineering calculations of the contact temperature.

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19 September 1967 Rostov-on-Don